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# DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE -- APRIL, 2019 <br> ENGINEERING MATHEMATICS - I 

[Time : 3 hours
(Maximum marks : 100)

## PART - A

(Maximum marks : 10)

I Answer all questions. Each question carries 2 marks.

1. Prove that $\cos ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~A}=1-2 \sin ^{2} \mathrm{~A}$.
2. If $\cos A=\frac{4}{5}$ and $A$ is acute, find $\cos 3 A$.
3. Find the area of the triangle $A B C$, given $b=3 \mathrm{~cm}, \mathrm{c}=2 \mathrm{~cm}, \mathrm{~A}=30^{\circ}$.
4. If $y=x \cos x$, Find $\frac{d y}{d x}$.
5. Find the velocity and acceleration at time ' $t$ ' of a particle moving according to $s=t^{3}-2 t^{2}+1$.

## PART — B

(Maximum marks : 30)
II Answer any five of the following questions. Each question carries 6 marks.

1. Express $3 \cos x+4 \sin x$ in the form $R \sin (x+\alpha)$, where $\alpha$ is acute.
2. Prove that $\cos \frac{\pi}{8}+\cos \frac{3 \pi}{8}+\cos \frac{5 \pi}{8}+\cos \frac{7 \pi}{8}=0$.
3. Prove that $(a+b) \sin \frac{c}{2}=c \cos \frac{A-B}{2}$.
4. Differentiate $\cos x$ by the method of first principles.
5. Find $\frac{d y}{d x}$ if $x^{3}+y^{3}=3 a x y$.
6. Find the equation to the tangent and normal to the curve $\mathrm{x}^{2+} \mathrm{y}^{2}=25$ at $(3,-4)$.
7. Prove that $\sin 120^{\circ} \cos 330^{\circ}+\cos 240^{\circ} \sin 330^{\circ}=1$.

## PART - C

(Maximum marks : 60)
(Answer one full question from each unit. Each full question carries 15 marks.)
UNIT - I

III (a) Prove that $\frac{\cos \theta}{1+\sin \theta}+\frac{1+\sin \theta}{\cos \theta}=2 \sec \theta$.
(b) If $\sin \mathrm{A}=\frac{3}{5}$ and A is acute, find $\sin 2 \mathrm{~A}$ and $\cos 2 \mathrm{~A}$.
(c) Show that $\tan 75^{\circ}+\cot 75^{\circ}=4$.

Or
IV (a) Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2 \sec ^{2} A$.
(b) If $\sin A=\frac{8}{17}, \sin B=\frac{3}{5} ; A, B$ are acute, find $\sin (A-B)$ and $\cos (A+B)$.
(c) From the top of a light house 90 m high, the angles of depression of two boats on the sea level are $45^{\circ}$ and $60^{\circ}$. Find the distance between the boats.

Unit - II
$V$ (a) Prove that $\frac{\sin 2 \mathrm{~A}+\sin 5 \mathrm{~A}-\sin \mathrm{A}}{\cos 2 \mathrm{~A}+\cos 5 \mathrm{~A}+\cos \mathrm{A}}=\tan 2 \mathrm{~A}$.
(b) Prove that $\cos 55^{\circ}+\cos 65^{\circ}+\cos 175^{\circ}=0$.
(c) Solve $\triangle \mathrm{ABC}$, given $\mathrm{a}=5 \mathrm{~cm}, \mathrm{~b}=8 \mathrm{~cm}$ and $\mathrm{C}=30^{\circ}$.

Or
VI (a) Prove that $\sin 50^{\circ}-\sin 70^{\circ}+\sin 10^{\circ}=0$.
(b) Prove that $\cos 3 \mathrm{~A}+\cos 5 \mathrm{~A}+\cos 9 \mathrm{~A}+\cos 17 \mathrm{~A}=4 \cos 4 \mathrm{~A} \cos 6 \mathrm{~A} \cos 7 \mathrm{~A}$.
(c) Solve $\triangle A B C$, given $a=2 \mathrm{~cm}, \mathrm{~b}=3 \mathrm{~cm}$ and $\mathrm{c}=4 \mathrm{~cm}$.

## Unit - III

VII (a) Evaluate (i) $\lim _{x \rightarrow 0} \frac{\sin 2 x \cdot \cos x}{x}$ (ii) $\lim _{x \rightarrow \infty} \frac{2 x^{2}+x+1}{x^{2}-2 x+1} \quad(3+3=6)$
(b) If $x=a \cos ^{3} \theta, y=b \sin ^{3} \theta$, find $\frac{d y}{d x}$.
(c) If $y=a \sin m x$, Prove that $\frac{d^{2} y}{d x^{2}}+m^{2} y=0$.

VIII (a) Evaluate (i) $\lim _{x \rightarrow 0} \frac{1-\cos x}{x^{2}} \quad$ (ii) $\lim _{x \rightarrow 1} \frac{x^{2}-1}{x-1} \quad(4+2=6)$
(b) If $\mathrm{y}=\log (\sec \mathrm{x}-\tan \mathrm{x})$, show that $\frac{\mathrm{dy}}{\mathrm{dx}}=-\sec \mathrm{x}$.
(c) If $y=A \sin x+B \cos x\left(A, B\right.$ are constants), Show that $\frac{d^{2} y}{d x^{2}}+y=0$

## Unit - IV

IX (a) Find the equations to the tangent and normal to the curve $y=3 x^{2}+x+2$ at ( 1,2 ).
(b) The radius of a circular plate is increasing in length at $0.1 \mathrm{~cm} / \mathrm{sec}$ when heated. What is the rate at which the area is increasing when the radius is 12 cm ?
(c) An open box is to be made out of a square sheet of side 18 cm by cutting off equal squares at each comer and tuming up the sides. What size of the squares should be cut in order that the volume of the box may be maximum?

## Or

X (a) Find the velocity and acceleration of a particle at $\mathrm{t}=4$ seconds whose displacement is given by $S=\frac{1}{2} t^{2}+\sqrt{t}$.
(b) A circular patch of oil spreads out on water, the area growing at the rate of $6 \mathrm{~cm}^{2}$ per minute. How fast is the radius increasing when the radius is 2 cms ?
(c) Find the maximum value of $2 x^{3}-9 x^{2}+12 x+5$.

