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## DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE - OCTOBER, 2017

## ENGINEERING MATHEMATICS - I

[Time : 3 hours
(Maximum marks : 100)

PART - A
(Maximum marks : 10)

## Marks

I Answer all questions. Each question carries 2 marks.

1. Prove that $(1+\cos \mathrm{A})(1-\cos \mathrm{A})=\sin ^{2} \mathrm{~A}$
2. Find the value of $3 \sin 15^{\circ}-4 \sin ^{3} 15^{\circ}$
3. Find $\frac{d y}{d x}$ if $y=x^{3} \tan x$.
4. Find the rate of change of volume $V$ with respect to the side of a cube.
5. Find the area of triangle ABC given $\mathrm{B}=3 \mathrm{~cm}, \mathrm{C}=2 \mathrm{~cm}$ and $\mathrm{A}=30^{\circ}$

> PART - B
(Maximum marks : 30)
II Answer any five of the following questions. Each question carries 6 marks.

1. Prove that $\left(\frac{\tan \theta+\sec \theta-1}{\tan \theta-\sec \theta+1}\right)=\frac{1+\sin \theta}{\cos \theta}$
2. If $\tan A=\frac{m}{m+1}, \tan B=\frac{1}{2 m+1} \quad A$ and $B$ are acute angles. Prove that $\mathrm{A}+\mathrm{B}=45^{\circ}$
3. Prove that $\sin 20^{\circ} \cdot \sin 40^{\circ} \cdot \sin 80^{\circ}=\frac{\sqrt{3}}{8}$
4. Prove that $R\left(a^{2}+b^{2}+c^{2}\right)=a b c(\cot A+\cot B+\cot C)$ where $R$ is radius of circumcircle.
5. Differentiate $x^{n}$ by method of first principles.
6. A particle moves such that the displacement from a fixed point ' $o$ ' is always given by $\mathrm{S}=5 \cos (\mathrm{nt})+4 \sin (\mathrm{nt})$ where n is a constant. Prove that the acceleration varies as its displacement S at the instant.
7. Find the equation to the tangent and normal to the curve $y=3 x^{2}+x-2$ at $(1,2)$.
PART - C
(Maximum marks : 60)
(Answer one full question from each unit. Each full question carries 15 marks.)
UNIT - I

III (a) Prove that $\left(\frac{1+\sin A}{\cos A}\right)=\left(\frac{\cos A}{1-\sin A}\right)$
(b) Prove that $\frac{\cos (90+A) \sec (360+A) \tan (180-A)}{\sec (A-720) \sin (540+A) \cot (A-90)}=1$
(c) If $\sin A=\frac{-4}{5}$ and $A$ lies in third quadrant, find all other trigonometric functions.

## Or

IV (a) If $\cos A=3 / 5, \tan B=5 / 12, A$ and $B$ are acute angles, find the values of $\sin (A+B)$ and $\cos (A-B)$.
(b) Prove that $\frac{\tan 45-\tan 30}{1+\tan 45 \tan 30}=2-\sqrt{3}$
(c) Express $5 \sin x-12 \cos x$ in the form $R \sin (x-\infty)$
UNIT - II

V (a) Prove that $\sin 33+\cos 63=\cos 3$
(b) Show that $(a-b) \cos \frac{C}{2}=c \sin \frac{A-B}{2}$
(c) Solve triangle ABC , given $\mathrm{a}=2 \mathrm{~cm} \mathrm{~b}=3 \mathrm{~cm} \mathrm{c}=4 \mathrm{~cm}$

Or
VI (a) Prove that $\cos \frac{\pi}{8}+\cos \frac{3 \pi}{8}+\cos \frac{5 \pi}{8}+\cos \frac{7 \pi}{8}=0$
(b) Prove that $2[b c \cos \mathrm{~A}+\mathrm{cacos} \mathrm{B}+\mathrm{ab} \cos \mathrm{C}]=\mathrm{a}^{2}+\mathrm{b}^{2}+\mathrm{c}^{2}$
(c) Two angles of a triangular plot of land are $53^{\circ} 17^{\prime}$ and $67^{\circ} 9^{\prime}$ and the side between them is measured to be 150 m . How many metres of fencing is required to fence the plot ?
UNIT - III

VII (a) Evaluate $\operatorname{Lt}_{x \rightarrow 0} \frac{\sqrt{(1+x)}-1}{x}$
(b) Find $\frac{d y}{d x}$, if
(i) $y=\frac{\cot 11 x}{\left(x^{3}-1\right)^{2}}$
(ii) $\left(x^{2}+1\right)^{10} \sec ^{5} \mathrm{x}$
(c) If $x=a(\theta+\sin \theta) y=a(1-\cos \theta)$ find $\frac{d y}{d x}$

Marks

VIII (a) Find the derivative of cotx using quotient rule.
(b) If $y=\sin ^{-1} x$ prove that $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}=0$
(c) If $x$ and $y$ are connected by the relation $a x^{2}+2 h x y+b y^{2}=0$ find $\frac{d y}{d x}$.
UNIT - IV

IX (a) Show that all the points on the curve $x^{3}+y^{3}=3 a x y$ at which the tangents are parallel to the $x$-axis lie on the curve, ay $=x^{2}$.
(b) A spherical balloon is inflated by pumping 25 cc of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm .
(c) The deflection of a beam is given by $y=4 x^{3}+9 x^{2}-12 x+2$. Find the maximum deflection.

## Or

X (a) Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square.
(b) A circular patch of oil spreads out on water, the area growing at the rate of $6 \mathrm{sq} . \mathrm{cm}$ per minute. How fast is the radius increasing when the radius is 2 cms .?
(c) The distance travelled by a moving body is given by $\mathrm{S}=2 \mathrm{t}^{3}-9 \mathrm{t}^{2}+12 \mathrm{t}+6$. Find the time when the acceleration is zero.

